

Comparing Commutation schemes for a Sinusoidal Motor

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Assumptions Regarding the Motor

Figure 1 shows an idealized diagram of a single pole pair brushless DC motor. Each of the three phases (or windings) is modeled as a simple series resistance, inductance, and magnetically-generated EMF, although we only consider the resistance and EMF in this analysis. Plots showing what the winding EMF's look like are shown in Appendix A.

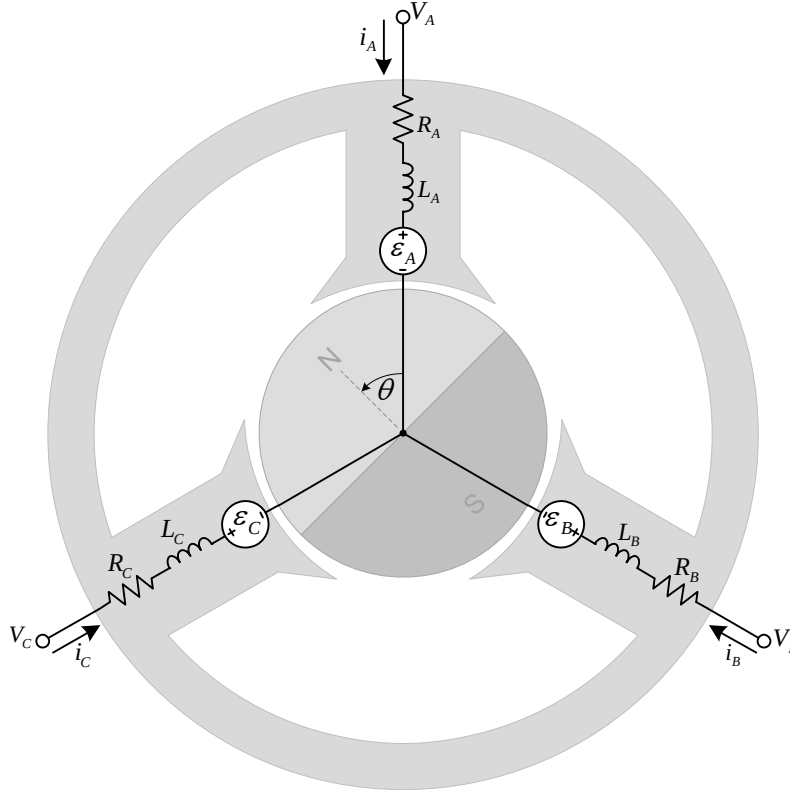


Figure 1. Diagram of a brushless DC motor

For all these cases, we assume the individual winding EMF's to be of the form:

$$\epsilon_A = k_v \dot{\theta} \sin(\theta) \quad 1.1$$

$$\epsilon_B = k_v \dot{\theta} \sin(\theta + 120^\circ) \quad 1.2$$

$$\epsilon_C = k_v \dot{\theta} \sin(\theta + 240^\circ) \quad 1.3$$

And the individual winding torques are given by:

$$T_A = k_t i_A \sin(\theta) \quad 1.4$$

$$T_B = k_t i_B \sin(\theta + 120^\circ) \quad 1.5$$

$$T_C = k_t i_C \sin(\theta + 240^\circ) \quad 1.6$$

The total torque produced by the motor (disregarding internal friction) is the sum of these:

$$\begin{aligned}
T &= T_A + T_B + T_C \\
&= k_t \left[i_A \sin(\theta) + i_B \sin(\theta + 120^\circ) + i_C \sin(\theta + 240^\circ) \right]
\end{aligned} \tag{1.7}$$

Constant-Current Trapezoidal Commutation

Assume the motor controller is doing whatever it needs to do to make the three winding currents of the form:

$$i_A = \begin{cases} +i_0 & ; \quad 30^\circ \leq \theta < 150^\circ \\ 0 & ; \quad 150^\circ \leq \theta < 210^\circ, 330^\circ \leq \theta < 390^\circ \\ -i_0 & ; \quad 210^\circ \leq \theta < 330^\circ \end{cases} \tag{1.8}$$

$$i_B = \begin{cases} +i_0 & ; \quad -90^\circ \leq \theta < 30^\circ \\ 0 & ; \quad 30^\circ \leq \theta < 90^\circ, 210^\circ \leq \theta < 270^\circ \\ -i_0 & ; \quad 90^\circ \leq \theta < 210^\circ \end{cases} \tag{1.9}$$

$$i_C = \begin{cases} +i_0 & ; \quad 150^\circ \leq \theta < 270^\circ \\ 0 & ; \quad 90^\circ \leq \theta < 150^\circ, 270^\circ \leq \theta < 330^\circ \\ -i_0 & ; \quad -30^\circ \leq \theta < 90^\circ \end{cases} \tag{1.10}$$

The average torque produced over one of the six commutation states is given by (using 1.7)¹:

$$\hat{T} = \frac{3}{\pi} \int_{30^\circ}^{90^\circ} T(\theta) d\theta = \frac{3\sqrt{3}k_t i_0}{\pi} \tag{1.11}$$

Note that brushless DC motors are normally marketed with the assumption of being driven using trapezoidal commutation. For this reason, motor datasheets specify an “average torque constant” which is related to the individual winding torque constant, k_t , by:

$$\hat{k}_t = \frac{3\sqrt{3}}{\pi} k_t \tag{1.12}$$

Rewriting the average motor torque in terms of this average torque constant²:

$$\hat{T} = \hat{k}_t i_0 \tag{1.13}$$

Sinusoidal Commutation with a sinusoidal motor

Assume the motor controller is doing whatever it needs to do to make the three winding currents of the form:

$$i_A = I \sin(\theta) \tag{1.14}$$

$$i_B = I \sin(\theta + 120^\circ) \tag{1.15}$$

$$i_C = I \sin(\theta + 240^\circ) \tag{1.16}$$

Plugging these into 1.7, the total torque produced is:

¹ See Appendix D for a step-by-step integration

² This explains why $\hat{k}_t$ is the reported parameter on motor datasheets. Simply multiplying it by the motor current yields average torque.

$$\begin{aligned}
T &= k_t I \left[\sin^2(\theta) + \sin^2(\theta + 120^\circ) + \sin^2(\theta + 240^\circ) \right] \\
&= \frac{3}{2} k_t I
\end{aligned}
\tag{1.17}$$

where we've used the identity:

$$\sin^2(\theta) + \sin^2(\theta + 120^\circ) + \sin^2(\theta + 240^\circ) = \frac{3}{2}
\tag{1.18}$$

Equation 1.17 expresses the relationship between torque and the amplitude of the individual winding currents in the motor, and also indicates that this torque is not a function of rotor position when using sine commutation (i.e. no torque ripple).

We can plug 1.12 into 1.17 to express the total torque in terms of the average torque constant:

$$T = \frac{\pi}{2\sqrt{3}} \hat{k}_t I
\tag{1.19}$$

Relating the Commutation Schemes

We can use total output torque (or at least the average of it) to compare these two commutation schemes. Equating (average) torque for sine and trapezoidal commutation (equations 1.13 and 1.19), we determine the equivalent trapezoidal current, i_0 , that is needed to produce the same amount of average torque that would be produced by a sinusoidal current of I :

$$\begin{aligned}
\frac{\pi}{2\sqrt{3}} \hat{k}_t I &= \hat{k}_t i_0 \\
\Rightarrow i_0 &= \frac{\pi}{2\sqrt{3}} I
\end{aligned}
\tag{1.20}$$

Appendices

Appendix A

Figure 1 shows a plot of the individual winding EMF's (dashed-line traces) and also the phase-to-phase EMF's (solid-line traces). The phase-to-phase EMF's represent the differential open-circuit voltages present at the motor winding wires when the motor shaft is driven by some external means at a fixed speed. For example, the V_{AB} trace represents the voltage measured from phase A to phase B as the motor is spun in a positive direction.

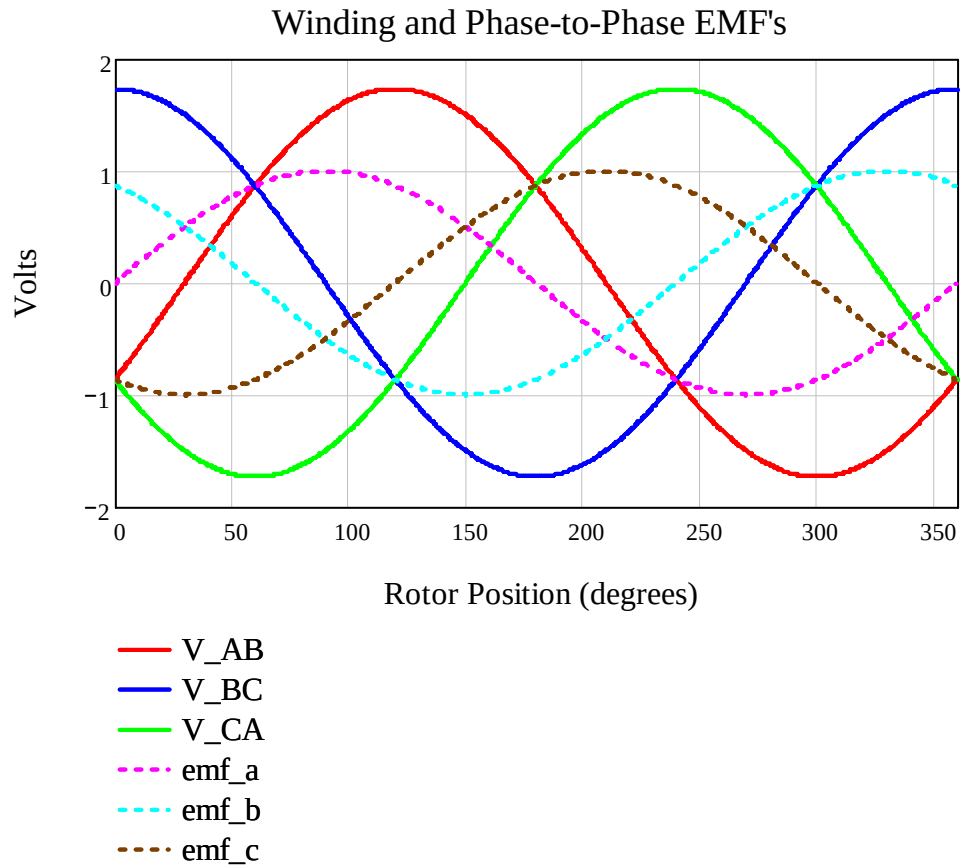


Figure 2. EMF plot

Appendix B

Figure 3 shows plots of individual winding currents for various commutation schemes. (Note that all currents are in reality centered at zero, but they have been vertically offset on the graph for readability.) The first three traces (i_{a_trap} , i_{b_trap} , i_{c_trap}) correspond to the winding currents for the trapezoidal commutation scheme considered in the above analysis. The next three traces (i_{a_sine} , i_{b_sine} , i_{c_sine}) correspond to sinusoidal commutation. The last three traces ($i_{a_trap_v}$, $i_{b_trap_v}$, $i_{c_trap_v}$) correspond to another type of trapezoidal commutation not considered in this analysis, whereby a constant voltage is applied across the motor windings during each of the six commutation states instead of a constant current. The total torque produced by the motor is the same for all three commutation schemes.

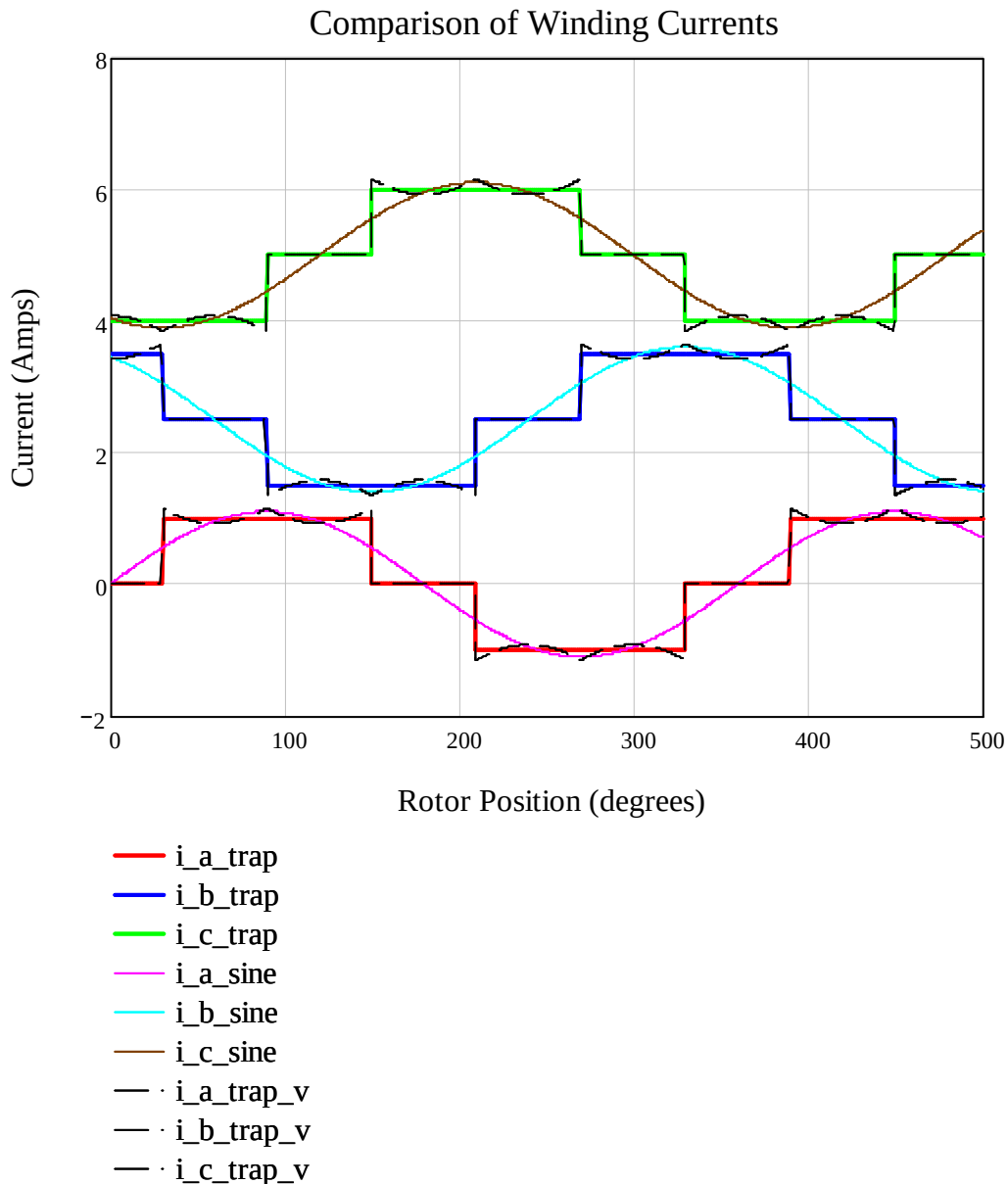


Figure 3. Plot showing winding currents for various commutation schemes

Appendix C

Figure 4 shows a plot of the total torque produced for various commutation schemes. Note how the torque is constant in the case of sinusoidal commutation, and how it varies as a function of rotor position for trapezoidal commutation (i.e. torque ripple). In the case of constant-voltage trapezoidal commutation (T_{trap_v}), the torque profile also changes as a function of motor speed. At stall, it matches T_{trap} exactly. As the motor speed increases, its shape changes and eventually “inverts” to shape shown. For even higher speeds, the amplitude of the ripple increases. The torque profile for constant-current trap commutation (T_{trap}) is independent of speed. The relation in 1.20 was used while generating this graph, which is why the average torque for all three traces is the same.

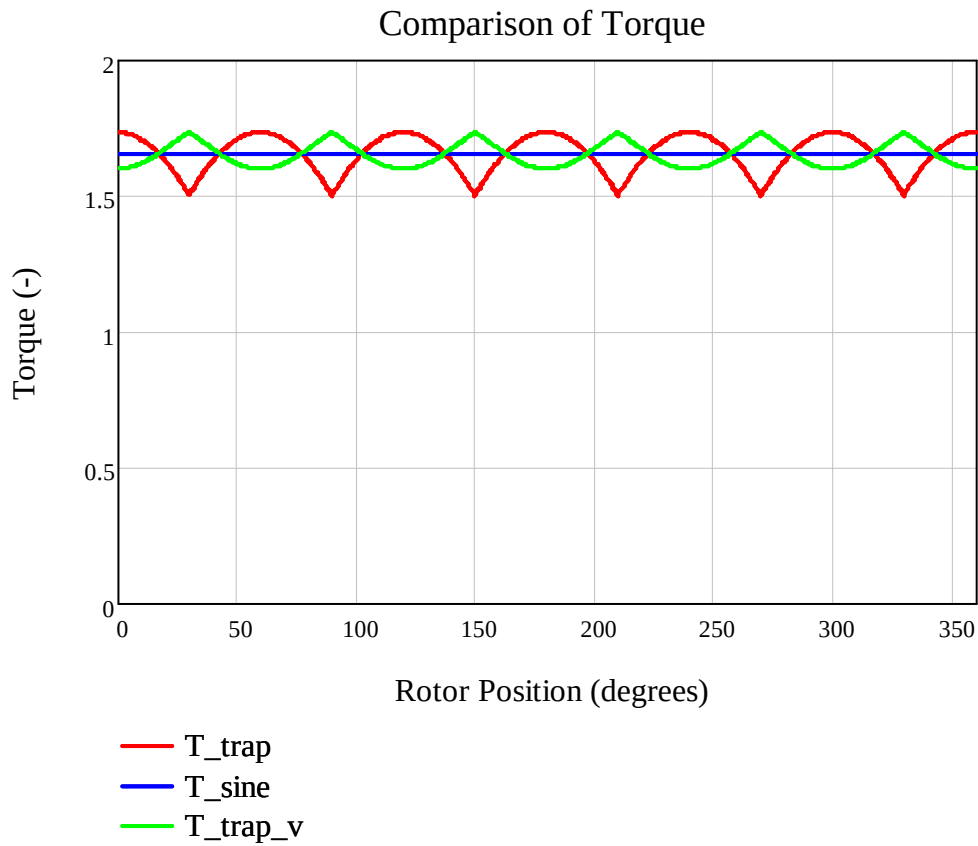


Figure 4. Plot showing total torque for various commutation schemes

Appendix D

Below is a detailed integration of equation 1.11.

$$\begin{aligned}\hat{T} &= \frac{3}{\pi} \int_{30^0}^{90^0} T_A + T_B + T_C d\theta \\&= \frac{3}{\pi} \int_{30^0}^{90^0} k_t i_A \sin(\theta) + k_t i_B \sin(\theta + 120^0) + k_t i_C \sin(\theta + 240^0) d\theta \\&= \frac{3}{\pi} \int_{30^0}^{90^0} k_t i_0 \sin(\theta) - k_t i_0 \sin(\theta + 240^0) d\theta \\&= \frac{3k_t i_0}{\pi} \int_{30^0}^{90^0} \sin(\theta) - \sin(\theta + 240^0) d\theta \\&= \frac{3k_t i_0}{\pi} \left[-\cos(\theta) + \cos(\theta + 240^0) \right]_{30^0}^{90^0} \\&= \frac{3k_t i_0}{\pi} \left[\cos(30^0) - \cos(90^0) + \cos(330^0) - \cos(270^0) \right] \\&= \frac{3k_t i_0}{\pi} \left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \right) \\&= \frac{3\sqrt{3}k_t i_0}{\pi}\end{aligned}$$